

Transformer Theory

one RV case

know x . $y=g(x)$ is monotonic

$$f_Y(y) = [f_X(x) \left| \frac{dx}{dy} \right|]_{x=g^{-1}(y)}$$

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many monotonic regions

$$f_Y(y) = \sum_{i \text{ region}} [f_X(x) \left| \frac{dx}{dy} \right|]_{x_i=z_i(y)}$$

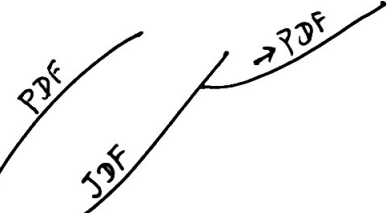
$$f_{Y_1, Y_2}(y_1, y_2) = [f_{X_1, X_2}(x_1, x_2) \cdot |J|]_{\substack{x_1=x_1(y_1, y_2) \\ x_2=x_2(y_1, y_2)}}$$

$$f_{Y_1, Y_2}(y_1, y_2) = [f_{X_1, X_2}(x_1, x_2) \cdot \frac{1}{|J^*|}]_{\substack{x_1=x_1(y_1, y_2) \\ x_2=x_2(y_1, y_2)}}$$

technique of using dummy

multiple transform

functions



requires f to be in exp. family
expression and special cases
requires support to depend on θ

Estimation

Finding the MVU estimator of θ (func of) n param. of a dist.

$$J = \begin{vmatrix} \frac{\partial x}{\partial y_1} & \frac{\partial x}{\partial y_2} \\ \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix}$$

$$J^* = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{vmatrix}$$

Approach 1: Cramer-Rao

Approach 2: Sufficient Stat.

Approach 3: Max-likelihood Theorem

Asymptotic Results

asymptotically unbiased

asymptotically normal dist.

asymptotically attaining CR bound

The form of θ that maximizes $\log f_{Y_1, \dots, Y_n}(y_1, \dots, y_n; \theta)$.

relation to sufficient statistics

MLE IS SS!

minimize w only fill mean is unbiased

From any unbiased estimator calc. the cond. prob.

From MNTSS to MVU estimator (Rao-Blackwell theorem)

$$E(\hat{T}(w))$$

how to find when support does depend on θ

when $y \in [A(\theta), B]$: $Y(w)$

when $y \in [A, B(\theta)]$: $Y(w)$

when $y \in [A(\theta), B(\theta)]$: difficult

definition - factorization-based expression
KKT unique: any one-to-one mapping on SS is also SS

MNTSS

what is

how to

find

minimal

exponential family

factorization + $\log(SS)$ should $< \infty$
non-triviality
Simplifies R